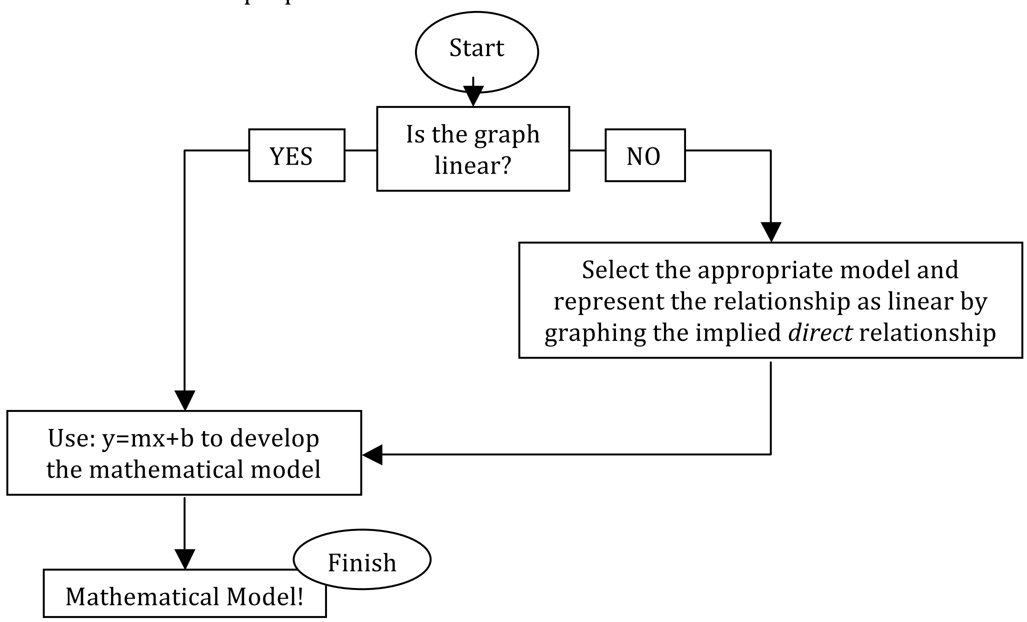
Developing a Mathematical Model

What do we do if after the graphing the data there is a clear relationship? We want to develop a mathematical model that is universal in its language and powerful in its ability to predict values beyond our data set (extrapolation) and within the data set (interpolation).

The development of the mathematical model follows directly from a graphical model, and is referred to as curve fitting. If the graphical model is linear, meaning that there is a constant ratio of the independent variable to dependent variable, the general equation is the y-axis slope intercept form $y = mx + b$. When this is the case, the development of the mathematical model is very simple, and the only calculation needed is that of the slope of the graph, and the determination of the y-intercept.

However if the graph under analysis is not linear, then we first must identify the relationship presented by the data, and then re-graph the data after manipulating it based on the direct proportionality suggested by the data. There are four graphical relationships that we will see in our study of motion and force in this class, this is only a small set of the possible mathematical models, but sufficient for our purposes.

In summary:



Two common mistakes when developing mathematical models:

1.) **Not completing the model**:

A mathematical model must communicate the relationship that is being studied. For example, if we are investigating the relationship between rain measured in inches, and tree growth measure in feet, and we find the equation of the linear graph to be: $y = 0.15\sqrt{x}$. If we stop here and keep the equation in this form it communicates nothing about the relationship we started out to investigate.

However, if I substitute the appropriate variables in for y (treegrowth) and x (rain), and <u>add units of measure</u> to the slope (0.15), the equation then becomes a mathematical model for the relationship that we are investigating.

Therefore the final mathematical model would look like this: $treegrowth = \left(.15\frac{feet}{\sqrt{inch}}\right) * \sqrt{rain}$

In summary:

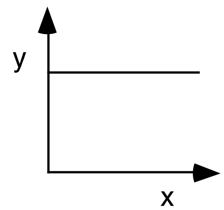
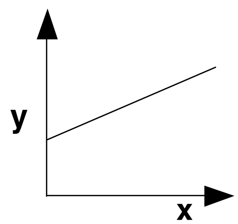
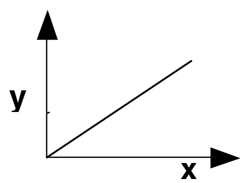
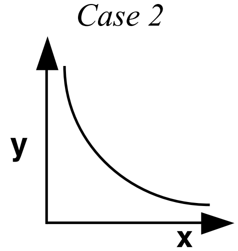
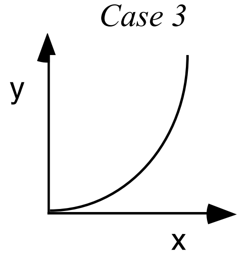
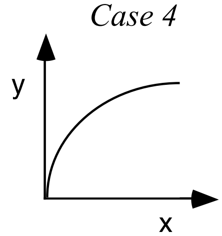
Not a Mathematical Model	A Mathematical Model
~~$y = 0.15\sqrt{x}$~~	$treegrowth = \left(.15\frac{feet}{\sqrt{inch}}\right) * \sqrt{rain}$

2.) **Selecting the incorrect model:**

Many students decided that the best mathematical model for the relationship between length and period is linear. This is an interesting proposal but presents a major logical flaw. In this model what is the period of a pendulum with no length? Remember, that we are looking for the *best* mathematical model for the relationship, not just the line that at first seems to fit the data set we have collected. *You must think about the implications of your model, and select the best model.*

Graphical Methods-Summary

A graph is one of the most effective representations of the relationship between two variables. The independent variable (one controlled by the experimenter) is usually placed on the x-axis. The dependent variable (one that responds to changes in the independent variable) is usually placed on the y-axis. It is important for you to be able interpret a graphical relationship and express it in a written statement and by means of an algebraic expression.

Graph shape	Written relationship	Modification required to linearize graph	Algebraic representation
Case 0 	As x increases, y remains the same. There is no relationship between the variables. They are independent of each other.	None	y is constant ($y = b$)
Case 1 	As x increases, y increases. Y is directly related to x.	None	$y = mx + b$ $y \propto x$
Case 1a 	As x increases, y increases proportionally. Y is directly proportional to x.	None	$y = mx$ $y \propto x$
Case 2 	As x increases, y decreases. Y is inversely proportional to x.	Graph y vs. $1/x$	$y = m\ 1/x + b$ $y \propto 1/x$
Case 3 	Y is proportional to the square of x.	Graph y vs. x^2	$y = mx^2 + b$ $y \propto x2$
Case 4 	The square of y is proportional to x.	Graph y^2 vs. x	$y^2 = mx + b$ $y2 \propto x$

Tools for Mathematical Modeling of Data

Uncertainty – the doubt that exists about a measurement in a quantitative form. A +/- number represents this doubt. There are two numbers (ave + unc; ave – unc).

Error – the difference between the measured value and the "true value" of the thing being measured. Whenever possible we try to correct for any known errors: for example by applying corrections we get from calibrations. Any error whose value we do not know is a source of uncertainty. Error and uncertainty are often used interchangeably.

Mistakes – mistakes made by you are not measurement uncertainties. These should not be counted as uncertainty but should be avoided by working carefully and by checking work.

Average – because there is variation in measurements it is best to take many readings and take an average. An average gives you an estimate of the "true" value. The symbol for average is x-bar, $\bar{x}$ an x with a line over it.

The 5-5-5 Rule – 5 trials – at least; 5 variations – change the variable 5 times (i.e. 5 angles, 5 masses); 5x (5 times) – separate the variations by at least 5 times the initial value – initial and final values separated by 5x (i.e. 10, 20, 30, 40, 50 sec and not 10, 10.1, 10.2, 10.3, sec)

Linear Regression – a procedure to get a line and equation that best fits your data (use the calculator!). Add units and variables to the equation. You can make predictions about values of the variables outside of your data set using this equation (generalizing).

Linearization – a procedure to allow you to get the simplest mathematical model for your data; y=mx+b

1. If the graph is linear (a straight line) use y = mx + b
2. If the graph is not linear
    a. Identify the relationship between the variables
        i. Direct relationship – *as x increases y increases*
        ii. Indirect relationship – *as x increases y decreases*
        iii. Direct proportional relationship – a ratio of the variables is constant. The line will go through zero. *x/y = a constant*
        iv. Inverse proportional relationship – a ratio of y to 1/x is constant *xy = a constant*
        v. Linear – the change in y over the change in x (slope) is constant;
            $\Delta y/\Delta x$ = *constant* or $y_2 - y_1/x_2 - x_1$ = *a constant*
    b. Re-graph by manipulating the variables to get a linear graph. The data doesn't change just the representation of it. Use the cheatsheet *Graphical Methods-Summary*!
3. Some of the Mathematical model equation's numbers may be very, very, very small as compared to the variables, which means essentially they are zero.

    Ex: trainspeed^2=(333m^2/s^3)time + 1x10^{-12}(m/s)^2 *can be seen as* trainspeed^2=(333 m^2/s^3)time